AP Calculus AB
Logarithmic Differentiation Worksheet

Name Key

Find the derivative of each function using logarithmic differentiation.

1. $y=(3x-7)^4(8x^2-1)^3$

$\ln y = 4\ln(3x-7) + 3\ln(8x^2-1)$

$\frac{1}{y}\frac{dy}{dx} = 4\cdot\frac{3}{3x-7} + 3\cdot\frac{16x}{8x^2-1}$

$\frac{1}{y}\frac{dy}{dx} = \frac{12}{3x-7} + \frac{48x}{8x^2-1}$

$\frac{dy}{dx} = \left(\frac{12}{3x-7} + \frac{48x}{8x^2-1}\right)(3x-7)^4(8x^2-1)^3$

2. $y=x^{2/5}(x^2+8)e^{x^2+x}$

$\ln y = \frac{2}{5}\ln x + \ln(x^2+8) + (x^2+x)\ln e$

$\ln y = \frac{2}{5}\ln x + \ln(x^2+8) + x^2 + x$

$\frac{1}{y}\frac{dy}{dx} = \frac{2}{5}\cdot\frac{1}{x} + \frac{2x}{x^2+8} + 2x + 1$

$\frac{dy}{dx} = \left(\frac{2}{5x} + \frac{2x}{x^2+8} + 2x + 1\right)x^{2/5}(x^2+8)e^{x^2+x}$

3. $y=\dfrac{(x+1)^4(x-5)^3}{(x-3)^8}$

$\ln y = 4\ln(x+1) + 3\ln(x-5) - 8\ln(x-3)$

$\frac{1}{y}\frac{dy}{dx} = 4\cdot\frac{1}{x+1} + 3\cdot\frac{1}{x-5} - 8\cdot\frac{1}{x-3}$

$\frac{dy}{dx} = \left(\frac{4}{x+1} + \frac{3}{x-5} - \frac{8}{x-3}\right)\frac{(x+1)^4(x-5)^3}{(x-3)^8}$

4. $y=\sqrt{\dfrac{x^2+1}{x+1}}$

$y = \frac{(x^2+1)^{1/2}}{(x+1)^{1/2}}$

$\ln y = \frac{1}{2}\ln(x^2+1) - \frac{1}{2}\ln(x+1)$

$\frac{1}{y}\frac{dy}{dx} = \frac{1}{2}\cdot\frac{2x}{x^2+1} - \frac{1}{2}\cdot\frac{1}{x+1}$

$\frac{dy}{dx} = \left(\frac{x}{x^2+1} - \frac{1}{2(x+1)}\right)\frac{(x^2+1)^{1/2}}{(x+1)^{1/2}}$

5. $y = x^x$

$\ln y = x \ln x$

$\frac{1}{y}\frac{dy}{dx} = (1)\ln x + (x)\left(\frac{1}{x}\right)$

$\frac{dy}{dx} = (\ln x + 1)\, x^x$

6. $y = x^{1/x}$

$\ln y = \frac{1}{x}\ln x$

$\ln y = x^{-1}\ln x$

$\frac{1}{y}\frac{dy}{dx} = (-x^{-2})\ln x + (x^{-1})\left(\frac{1}{x}\right)$

$\frac{dy}{dx} = \left(-\frac{\ln x}{x^2} + \frac{1}{x^2}\right) x^{1/x}$

or

$\frac{dy}{dx} = \left(\frac{1-\ln x}{x^2}\right) x^{1/x}$

7. $y = x^{\sin x}$

$\ln y = \sin x \ln x$

$\frac{1}{y}\frac{dy}{dx} = (\cos x)\ln x + (\sin x)\left(\frac{1}{x}\right)$

$\frac{dy}{dx} = \left(\cos x \ln x + \frac{\sin x}{x}\right) x^{\sin x}$

8. $y = (\sin x)^x$

$\ln y = x \ln(\sin x)$

$\frac{1}{y}\frac{dy}{dx} = (1)\ln(\sin x) + (x)\frac{\cos x}{\sin x}$

$\frac{dy}{dx} = \left(\ln(\sin x) + x\cot x\right)(\sin x)^x$

9. $y = (\ln x)^x$

$\ln y = x \ln(\ln x)$

$\frac{1}{y}\frac{dy}{dx} = (1)\ln(\ln x) + (x)\left(\frac{\frac{1}{x}}{\ln x}\right)$

$\frac{dy}{dx} = \left(\ln(\ln x) + \frac{1}{\ln x}\right)(\ln x)^x$

10. $y = x^{\ln x}$

$\ln y = \ln x \cdot \ln x$

$\frac{1}{y}\frac{dy}{dx} = \left(\frac{1}{x}\right)\ln x + \ln x\left(\frac{1}{x}\right)$

$\frac{dy}{dx} = \left(\frac{2\ln x}{x}\right) x^{\ln x}$

$y = (\ln x)^2$

$\frac{1}{y}\frac{dy}{dx} = 2(\ln x)^1\left(\frac{1}{x}\right)$

$\frac{dy}{dx} = \left(\frac{2\ln x}{x}\right) x^{\ln x}$

11. Find $\frac{dy}{dx}$ if $y = \ln(x^2 + y^2)$

$\frac{dy}{dx} = \frac{2x + 2y\frac{dy}{dx}}{x^2+y^2}$

$(x^2+y^2)\frac{dy}{dx} = 2x + 2y\frac{dy}{dx}$

$(x^2+y^2)\frac{dy}{dx} - 2y\frac{dy}{dx} = 2x$

$\frac{dy}{dx} = \frac{2x}{x^2+y^2-2y}$